1. \[(a-2)^4 = \frac{a^{-8}}{a} = a^{-9} = 1 \quad \text{(applying } (a^m)^n = a^{mn}) \quad \text{(1 Mark)}\]

\[a = \frac{1}{a^9} \quad \text{(applying } \frac{a^m}{a^n} = a^{m-n}) \quad \text{(1 Mark)}\]

\[= 1 \quad \text{(1 Mark)}\]

2. \[y(5 + x) = 4 - x \quad \text{(1 Mark)}\]

\[5y + xy = 4 - x \quad \text{(1 Mark)}\]

\[(y + 1)x = 4 - 5y \quad \text{(1 Mark)}\]

\[x = \frac{4 - 5y}{1 + y} \quad \text{(1 Mark)}\]

3. \[\text{Perimeter of the sector} \]

\[= 2\pi(4) \times \frac{50}{360} + 2(4) \text{ cm} \quad \text{(2 Mark)}\]

\[= \left( \frac{10\pi}{9} + 8 \right) \text{ cm} \quad \text{(1 Mark)} \]

\[\text{Figure 1:} \quad \text{accept 11.5 cm or r.t.}\]

4. \[\text{Figure 2:} \quad a = \sqrt{11^2 - 8^2} \quad \text{(1 Mark)} \]

\[= \sqrt{57} \quad \text{(1 Mark)} \]

\[\cos x^o = \frac{8}{11} = 0.72727272 \quad \text{(1 Mark)} \]

\[x = 43.3 \quad \text{(1 Mark)} \]

\[\text{u-1 for } 43.3^o \]

5. \[\angle ADC = 90^o \quad \text{in semi-circle} \]

\[x + 25 + 90 = 180 \quad \text{sum of } \triangle ADC \]

\[x = 65 \]

\[\angle ADB = 55^o \quad \text{s in the same segment} \]

\[y = 90 - 55 = 35 \]

\[\text{Figure 3:} \]

6. \[11 - 2x \geq 3 \quad \text{(1 Mark)} \]

\[8 \geq 2x \quad \text{(1 Mark)} \]

\[x \leq 4 \quad \text{(1 Mark)} \]

\[\text{for correct graphical solution} \]
7. (a) Selling price
\[ \text{Selling price} = 270 \times (1 - 25\%) = 202.5 \]
(b) Percentage profit
\[ \text{Percentage profit} = \frac{202.5 - 180}{180} \times 100\% = 12.5\% \]
7. (b) Percentage profit
\[ \text{Percentage profit} = \frac{202.5 - 180}{180} \times 100\% = 12.5\% \]

8. \[ y = A + Bx^3 \] where A and B are constants
\[ \begin{align*}
19 &= A + 8B \\
57 &= A + 27B
\end{align*} \]
Solving, \( B = 2 \) and \( A = 3 \)
Therefore, \( y = 3 + 2x^3 \)

9. \[ c = 8 \]
Therefore, \( y = -x^2 + 2x + 8 \)
Putting \( 0 = -x^2 + 2x + 8 \)
\( (x - 4)(x + 2) = 0 \)
\( x = 4 \) or \(-2 \)
Therefore, \( a = -2 \) and \( b = 4 \)

10. (a) Let \( Vm^3 \) be the capacity of the pool. Then the filling rates of pipe A, B and C are \( \frac{V}{x + 5} \) m\(^3\)/hour, \( \frac{V}{x + 2} \) m\(^3\)/hour and \( \frac{V}{x} \) m\(^3\)/hour respectively.
\[ \frac{1}{x + 5} + \frac{1}{x + 2} + \frac{1}{x} = \frac{4}{V} \]
\[ \frac{3x^2 + 14x + 10}{x^3 + 7x^2 + 10x} = \frac{1}{4} \]
\[ x^3 - 5x^2 - 46x - 40 = 0. \]
(b) \[ x^3 - 5x^2 - 46x - 40 = 0 \]
\[ = (x + 1)(x^2 - 6x - 40) \]
\[ = (x + 1)(x + 4)(x - 10) \]
Therefore, for \[ x^3 - 5x^2 - 46x - 40 = 0 \]
\[ x = -1, -4 \text{ or } 10 \]
By the nature of the problem, the first two roots are rejected.
Hence \[ x = 10 \].

11. (a)

<table>
<thead>
<tr>
<th>Test score (x)</th>
<th>Class mid-value</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 &lt; x ≤ 50</td>
<td>45</td>
<td>10</td>
</tr>
<tr>
<td>50 &lt; x ≤ 60</td>
<td>55</td>
<td>46</td>
</tr>
<tr>
<td>60 &lt; x ≤ 70</td>
<td>65</td>
<td>52</td>
</tr>
<tr>
<td>70 &lt; x ≤ 80</td>
<td>75</td>
<td>54</td>
</tr>
<tr>
<td>80 &lt; x ≤ 90</td>
<td>85</td>
<td>30</td>
</tr>
<tr>
<td>90 &lt; x ≤ 100</td>
<td>95</td>
<td>8</td>
</tr>
</tbody>
</table>

(b) mean = 68.6
standard deviation = 12.4

(c) inter-quartile range
\[ = 77.8 - 58.7 \]
\[ = 19.1 \]

12. (a) i. Perimeter of \( G_{50} \)
\[ = [15 + (50 - 1) \times 1] \text{ cm} \]
\[ = 64 \text{ cm} \]

ii. Sum of the perimeters
\[ = \left[ 50 \times \left( \frac{15 + 64}{2} \right) \right] \text{ cm} \]
\[ = 1975 \text{ cm} \]

(b) i. Area of \( G_2 \)
\[ = \left[ 5 \times \left( \frac{16}{15} \right)^2 \right] \text{ cm}^2 \]
\[ = \frac{256}{45} \text{ cm}^2 \]

ii. Area of \( G_3 \)
\[ = \left[ 5 \times \left( \frac{17}{15} \right)^2 \right] \text{ cm}^2 \]
\[ = \frac{289}{45} \text{ cm}^2 \]

Area of \( G_2 \) - Area of \( G_1 \neq \) Area of \( G_3 \) - Area of \( G_2 \)
\[ \left( \frac{31}{45} \text{ cm}^2 \neq \frac{64}{45} \text{ cm}^2 \right) \]
Therefore, the area of the figures \( G_1, G_2, G_3, \ldots, G_{50} \) do not form an arithmetic sequence.

13. (a) Slope of \( BC = \frac{4 - 0}{1 - 2} = -4 \)
Question 13, continued  

F.5 Mathematics I  

Marking Scheme

(b) Slope of HP $= \frac{-1}{-4} = \frac{1}{4}$  
Equation of HP is $\frac{y-\frac{2}{3}}{x-\frac{1}{2}} = \frac{1}{4}$  
or $2x - 8y + 13 = 0$  

(c) i. The coordinates of H $= \left(0, \frac{13}{8}\right)$  
ii. Equation of the circle passing through A, B, C is $x^2 + y^2 - 2(0)x - 2\left(\frac{13}{8}\right) + k = 0$ for some constant k.  
By substituting B(2,0), $k = -\frac{16}{4}$.  
Required equation of circle is $4x^2 + 4y^2 - 13y - 16 = 0$

14.  

| AE = AC | given |  
| AD = AB | given |  
| DE = BC | given |  
| $\triangle AED \cong \triangle ACB$ | S.S.S |  
| $\angle DAE = \angle BAC$ | corr. $\angle$ of $\cong \triangle$s |  
| $\angle DAC = \angle DAE + \angle EAC$ | $= \angle BAC + \angle EAC$ |  
| $= \angle BAE$ |  
| AD = AB | given |  
| AC = AE | given |  
| $\triangle DAC \cong \triangle BAE$ | S.A.S |  
| BE = CD | corr. $\angle$ of $\cong \triangle$s |  

15. (a) Let $r$ cm be the radius of the surface of the melting ice-cream. By considering the volumes of the ice-cream,  
$\frac{4}{3}\pi \left(\frac{5}{2}\right)^3 + \frac{4}{3}\pi x^3 = \frac{1}{3}\pi r^2(2x + 3)$  
Using similar triangles,  
$r = \frac{5}{10} = \frac{1}{2}(2x + 3)$  
Volume of the liquid $= \frac{1}{3} \pi \left[\frac{1}{2}(2x + 3)\right]^2 (2x + 3)cm^3$  
$= \frac{1}{12} \pi (2x + 3)^3 cm^3$  
Hence $\frac{4}{3}\pi \left(\frac{5}{2}\right)^3 + \frac{4}{3}\pi x^3 = \frac{1}{3}\pi (2x + 3)^3$  
$250 + 16x^3 = (2x + 3)^3 = 8x^3 + 36x^2 + 54x + 27$  
Therefore, $8x^3 - 36x^2 - 54x + 223 = 0$
(b) From (a), 
\[8x^3 - 36x^2 - 54x + 223 = 0\]
\[4(2x^3 - 9x^2 + 13x) - 106x + 223 = 0\]
\[4y - 106x + 223 = 0\]
Adding the line in Figure 7,
we have \(x \approx 2.4\)

16. (a) \(OA' = 2 \tan 50^\circ \) (m)
\(OB' = 2.7 \tan 50^\circ \) (m)
\(A'B' = OB' - OA'\)
\(= 0.7 \tan 50^\circ \) (m)
\(= 0.834227514 \text{ m}\)  
1A r.t. 0.834m

(b) \(\cos \angle BAC = \frac{0.7^2 + 0.8^2 - 0.9^2}{2 \times 0.7 \times 0.8}\)
\(\angle BAC = 73.984504^\circ\)
\(OD = AC \sin \angle BAC = 0.766651878 \text{ m}\)  
1A r.t. 0.767m

(c) Area of the shadow \(A'B'C'\)
\[= \frac{1}{2} A'B' \times OD\]
\[= \frac{1}{2} \times 0.834227514 \times 0.766651878 \text{ m}^2\]
\[= 0.319781045 \text{ m}^2\]  
1A r.t. 0.320 \text{ m}^2

(d) i. Shadow of \(AB\) is longer than \(A'B'\) in (a)  
as \(A'B' = AB / \tan(\text{angle of elevation})\) and tangent is increasing from 0\(^\circ\) to 90\(^\circ\)  
1A

ii. Area of the shadow of the road sign \(ABC\) is larger than that of \(A'B'C'\) in (c) because area of shadow \(A'B'C' = \frac{1}{2} A'B' \times OD\).  
1A
17. (a) as shown in Figure 8. 

(b) 

i. as shown in Figure 9.

ii. \( \alpha \) Required probability \( = \frac{34}{72} = \frac{17}{36} \)

\( \beta \) Required probability \( = \frac{34}{72} \times \frac{11}{71} + \frac{11}{72} \times \frac{34}{71} \)

\( = \frac{187}{1278} \)

18. (a) 

i. \( \text{TH} = \text{TA} \) 

\( \text{TA} = \text{TK} \) properties of ext. tangents 

\( \text{TH} = \text{TK} \)

ii. \( \text{TA} \perp \text{AB} \) tangent \( \perp \) radius

\( \text{T is the centre of circle HAK by (a)(i)} \)

AB touches the circle HAK

(b) 

i. \( \text{HK} = \sqrt{(a + b)^2 - (a - b)^2} \)

\( = 2\sqrt{ab} \)

1A

ii. Coordinates of \( T = \left( a, \sqrt{ab} \right) \)

1A

iii. Let \( \theta \) be the acute angle between HK and the x-axis. (Refer to Figure 10.)

\( \cos \theta = \frac{2\sqrt{ab}}{a + b} \)

\( \sin \theta = \frac{a - b}{a + b} \)

Coordinates of \( K \)

\( = \left( a + TK \cos \theta, \sqrt{ab} - TK \sin \theta \right) \)

\( = \left( a + \sqrt{ab} \left( \frac{2\sqrt{ab}}{a + b} \right), \sqrt{ab} - \sqrt{ab} \left( \frac{a - b}{a + b} \right) \right) \)

\( = \left( \frac{a^2 + 3ab}{a + b}, \frac{2b\sqrt{ab}}{a + b} \right) \)

1A
The graph of $y = 2x^3 - 9x^2 + 13x$

$y = \frac{106}{4}x - \frac{223}{4}$
Figure 8:

Graph showing a line with the equation $7x - 2y = 21$ and points on the grid.

Figure 9:

Grid with shaded circles and another grid with unfilled circles.

Figure 10:

Geometric diagram involving circles and angles.