Question: 1 2 3 Total
Marks: 6 6 7 19
Score: 

1. (a) For all $x > 0$, find $\frac{d}{dx}(x^x)$. (3 marks)
   (b) Evaluate $\lim_{x \to 1} \frac{x^x - x}{\ln x - x + 1}$. (3 marks)

2. (a) Find $\lim_{x \to 0^+} x^2 \ln x$. (2 marks)
   (b) Let $k$ be a real constant and $f : \mathbb{R} \to \mathbb{R}$ be defined by
      
      $$f(x) = \begin{cases} 
      \sin 2x + \cos x + k & \text{when } x \leq 0, \\
      x^3 \ln x & \text{when } x > 0. 
      \end{cases}$$
      
      It is given that $f(x)$ is continuous at $x = 0$.
      (i) Find $k$. (2 marks)
      (ii) Is $f(x)$ differentiable at $x = 0$? Explain your answer. (2 marks)

3. Let $a$, $b$ and $c$ be real constants and $f : \mathbb{R} \to \mathbb{R}$ be defined by
   $$f(x) = \begin{cases} 
      \frac{12 + a\sqrt{9-x}}{x} & \text{when } x < 0, \\
      c + b \tan \frac{x}{8} & \text{when } x \geq 0. 
      \end{cases}$$
   
   It is given that $f(x)$ is continuous at $x = 0$.
   (a) Explain why $\lim_{x \to 0} f(x) = c$. (1 mark)
   (b) Hence find $a$ (1 mark)
   (c) Find $c$. (2 marks)
   (d) Furthermore, $f(x)$ is differentiable at $x = 0$. Find $b$. (3 marks)