Lemma 0.1 (Proof by Contradiction). Let $x$ be a non-real number and $a$, $b$, $c$, $d$ be real numbers. If $a + bx = c + dx$, then $a=c$ and $b=d$.

Example 0.2. Resolve \( \frac{x^2 + 1}{x^3 + 1} \) into partial fractions.

Solution:
Using Cover-up Rule for the linear factor \((x+1)\), we have
\[
\frac{x^2 + 1}{x^3 + 1} \equiv \frac{2}{(x+1)(x^2 - x + 1)} \equiv \frac{Ax + B}{x^2 - x + 1}.
\]
It is easy to see that \( x^2 - x + 1 = 0 \) has two unequal non-real roots. Let \( \alpha \) be one of them. Note that \( \alpha^2 = \alpha - 1 \). (Why?) Using the same trick for cover-up rule, we have
\[
A\alpha + B = \frac{\alpha^2 + 1}{\alpha + 1} = \frac{\alpha}{\alpha + 1} = \frac{\alpha(\alpha + 1)}{(\alpha + 1)^2} = \frac{\alpha(\alpha + 1)}{\alpha^2 + 2\alpha + 1} = \frac{3\alpha}{3}\frac{\alpha + 1}{3} = \frac{1}{3}\alpha + \frac{1}{3}.
\]
\[\therefore A = \frac{1}{3} = B.\]
Answer: \( \frac{x^2 + 1}{x^3 + 1} \equiv \frac{2}{3(x+1)} + \frac{x + 1}{3(x^2 - x + 1)}. \)
Remark: With a little training, one can meditate all this in one’s head and roll out the answer in just one line.

Example 0.3. Resolve \( \frac{x^3 + 1}{x^4 + 1} \) into partial fractions.
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