1. For $x \neq \sqrt[3]{2}$,

$$f(x) = \frac{|x|x^3}{x^3 - 2} = sgn(x) \frac{x(x^3 - 2 + 2)}{x^3 - 2}$$

$$f'(x) = sgn(x) \left[ x + \frac{2x}{x^3 - 2} \right] - \frac{2x(3x^2)}{(x^3 - 2)^2}$$

$$f''(x) = sgn(x) \left[ 1 + \frac{2}{x^3 - 2} - \frac{6x^2 - 12 + 12}{(x^3 - 2)^2} \right] = sgn(x) \left( 1 - \frac{6}{x^3 - 2} \right) \left( 1 + \frac{2}{x^3 - 2} \right)$$

$$f''(x) = \frac{sgn(x)(x^3 - 8)x^3}{(x^3 - 2)^2}$$

(a) i. For $x > 0$,

$$f(x) = \frac{(x^3 - 8)x^3}{(x^3 - 2)^2}$$

$$f''(x) = \frac{12x^2(x^3 + 4)}{(x^3 - 2)^3}$$

ii. For $x < 0$,

$$f(x) = -\frac{(x^3 - 8)x^3}{(x^3 - 2)^2}$$

$$f''(x) = -\frac{12x^2(x^3 + 4)}{(x^3 - 2)^3}$$

iii. $\lim_{x \to 0} \frac{f(x) - f(0)}{x} = \lim_{x \to 0} \frac{|x|x^2}{x^3 - 2} = 0$. \( \therefore \) $f'(0)$ exists and equal 0.

iv. $\lim_{x \to 0} \frac{f'(x) - f'(0)}{x} = \lim_{x \to 0} \frac{x|x|(x^3 - 2)}{(x^3 - 2)^2} = 0$. \( \therefore \) $f''(0)$ exists and equal 0.

(b) Find the range of $x$ for which $x$ satisfies the following:

i. $x > 2$.

ii. $x < \sqrt[3]{2}$ or $\sqrt[3]{2} < x < 2$.

iii. $-\sqrt[4]{4} < x < 0$ or $x > \sqrt[3]{2}$.

iv. $x < -\sqrt[4]{4}$ or $0 < x < \sqrt[3]{2}$.

(c) Local minimum point = \( \left( 2, \frac{8}{3} \right) \). There is no maximum point.

Inflexional points = \( \left( -\sqrt[4]{4}, \frac{2}{3}\sqrt[4]{4} \right), (0, 0) \)
(d) Vertical asymptote is \( x = \sqrt{2} \)
   Oblique asymptote is \( y = |x| \)

(e) \[
\begin{align*}
  x &= 3\sqrt{2} \\
  y &= \frac{|x|}{x^3 - 2}
\end{align*}
\]