General marking guidelines:

(a) In the marking scheme, marks are classified into ‘M’ marks for correct methods being used and ‘A’ marks for the accuracy of the answers.

(b) Marks may be deducted for wrong units (u) or poor presentation (pp). The symbol u-1 or pp-1 are used to denote 1 mark deducted for u or pp. At most 1 mark would be deducted in each of Section A(1) and Section A(2). No deduction for any marks for u or pp in Section B. At most 1 mark could be deducted in each of Section A(1) and Section A(2).

(c) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures. No ‘A’ marks or pp-1 would be awarded for numerical answers not satisfying the above requirement.

1.

\[
\frac{a^3 b^{-2}}{(a^2 b)^4} = \frac{a^3 b^{-2}}{a^{-8} b^4} = a^{3-(8)} b^{-2-4} = a^{11} b^{-6} = \frac{a^{11}}{b^6}
\]

1A

For \((ab)^k = a^{kb}\) or \((a^j)^k = a^{jk}\)

For \(\frac{c^n}{c^m} = c^{n-m}\) or \(c^{-m} = \frac{1}{c^m}\)

2.

\[
\frac{mk - t}{k + t} = 4
\]

1M

\[
mk - t = 4 (k + t)
\]

1M

\[
mk - t = 4k + 4t
\]

1M

for putting k on one side

\[
mk - 4k = 5t
\]

1M

\[
k (m - 4) = 5t
\]

1A

\[
k = \frac{5t}{m - 4}
\]

1A

3.

(a) \[9x^2 - 4y^2 = (3x)^2 - (2y)^2 = (3x - 2y)(3x + 2y)\]

1A

(b) \[9x^2 - 4y^2 - 4y - 6x = (3x - 2y)(3x + 2y) - 2(3x + 2y) = (3x + 2y)(3x - 2y - 2)\]

1A

for using (a)
4.
(a) 6090
(b) 6100
(c) 6091.180

5.
Let \( x \) be the original number of girls in the X’mas ball.
Then, the original number of boys in the X’mas ball is \( \frac{7x}{5} \)

\[
\frac{7x}{5} - 100 = x - 30
\]
\[
7x - 500 = 5x - 150
\]
\[
x = 175
\]
The number of boys is 245 and the number of girls is 175.
The original number of participants is 420.

6.
(a) \( \angle AOB = 162^\circ - 72^\circ = 90^\circ \)
\[\therefore AO \perp BO\]
(b) \[\therefore AO \perp BO\]
\[\therefore \text{Area of } \triangle AOB = \frac{1}{2}(6)(8) = 24 \text{ sq. units}\]

7.
(a) \[\therefore \triangle ABE \cong \triangle DCE\]
\[\therefore AE = DE \text{ and } BE = CE \text{ (corr. sides, } \cong \triangle s)\]
\[\therefore \triangle AED \text{ and } \triangle BEC \text{ are isosceles triangles.}\]
(b) \[\therefore \triangle BEC \text{ is an isosceles triangle,}\]
\[\therefore \angle BCE = \angle CBE = 25^\circ \text{ (base } \angle s, \text{ isos. } \triangle)\]
\[\angle ACD = 90^\circ \text{ (} \angle \text{ in semi - circle)}\]
\[\text{In } \triangle BCD, \]
\[\angle CDE + (90^\circ + 25^\circ) + 25^\circ = 180^\circ \text{ (} \angle \text{ sum of } \triangle)\]
\[\angle CDE = 40^\circ\]
8. \( \angle BAC = 220^\circ - 180^\circ = 40^\circ \)  

Let the required angle be \( x \)  
\[ x = \angle BAC = 40^\circ \text{ (alt. } \angle s, // \text{ lines) } \]  
\[ \therefore \] The true bearing of A from B is \( 040^\circ \).  

(b) \( \therefore AB \perp BC \)  
\[ \therefore \angle ABC = 90^\circ \]  
\[ \angle ACB = 180^\circ - \angle BAC - \angle ABC (\angle \text{ sum of } \Delta) \]  
\[ = 180^\circ - 40^\circ - 90^\circ \]  
\[ = 50^\circ \]  
\[ \therefore \] The compass bearing of B from C is N50°W.

9. 

(a) \[ \tan \theta = \frac{\cos \theta + 1}{\sin \theta} \]  
\[ \sin \theta = \frac{\cos \theta + 1}{\cos \theta} \]  
\[ \cos \theta = \frac{\sin \theta}{\sin \theta} \]  
\[ \sin^2 \theta = \cos^2 \theta + \cos \theta \]  
\[ 1 - \cos^2 \theta = \cos^2 \theta + \cos \theta \]  
\[ 2 \cos^2 \theta + \cos \theta - 1 = 0 \]  

(b) By the result of (a).  
\[ 2y^2 + y - 1 = 0 \text{ where } y = \cos \theta \]  
\[ (2y - 1)(y + 1) = 0 \]  
\[ \therefore \]  
\[ y = \frac{1}{2} \text{ or } -1 \]  
\[ \cos \theta = \frac{1}{2} \text{ or } \cos \theta = -1 \]  
\[ \theta = 60^\circ, 360^\circ - 60^\circ \text{ or } 180^\circ \text{ (rejected)} \]  
\[ = 60^\circ \text{ or } 300^\circ \]  

2A marks are still awarded if 180° is not rejected.
10. (a) Let \( z = C + k\sqrt{x} \) where \( C, k \) are constants.

\[
\begin{aligned}
27 &= C + k \times \sqrt{9} \\
30 &= C + k \times \sqrt{16}
\end{aligned}
\]

\[
\begin{aligned}
C + 3k &= 27 \quad \text{.................}(1) \\
C + 4k &= 30 \quad \text{.................}(2)
\end{aligned}
\]

(2) – (1):

\[4k - 3k = 30 - 27\]

\[k = 3\]

Substituting \( k = 3 \) into (1),

\[C + 3 \times 3 = 27\]

\[C = 18\]

\[\therefore z = 18 + 3\sqrt{x}\]

(b) When \( x = 36 \),

\[z = 18 + 3\sqrt{36}\]

\[= 18 + 18\]

\[= 36\]

11. (a) By comparing the coefficients of \( x^3 \) and the constant terms, we have \( a = 3 \) and \( c = 4 \).

Note that the coefficient of \( x^2 \) in the expansion of 

\[(x - 2)(3x^2 + bx + 4)\]

is \( b - 6 \).

By comparing the coefficients of \( x^2 \), we have \( b - 6 = -7 \).

Thus, we have \( b = -1 \).

or

Note that \( x - 2 \) is a factor of \( f(x) \).

\[f(2) = 0\]

\[= 3(2)^3 - 7(2)^2 + 6x - 8\]

\[= (x - 2)(3x^2 - x + 4)\]

Thus, we have \( a = 3, \ b = -1 \) and \( c = 4 \).

(b) \( \Delta \)

\[= (-1)^2 - 4(3)(4)\]

\[= -47 < 0\]

So, the equation \( (3x^2 - x + 4) = 0 \) has nonreal roots.

Thus, the claim is disagreed.
12. (a) \[ BD = \sqrt{8^2 + 8^2} \]
\[ = 8\sqrt{2} \text{ cm} \]
\[ BP = 8\sqrt{2} \times \frac{1}{2} \text{ (where } P \text{ is the projection of } V \text{ on } ABCD) \]
\[ = 4\sqrt{2} \text{ cm} \]
\[ VP \text{ is the height and } \angle VPB = 90^\circ \]
\[ VP^2 + (4\sqrt{2})^2 = 20^2 \]
\[ VP^2 = 20^2 - (4\sqrt{2})^2 = 368 \]
\[ VP = \sqrt{368} \]
\[ = 19.2 \text{ cm} \]
(b) Let \( Q \) be the mid-point of \( CD \),
\[ \angle VQP \text{ is the required angle.} \]
\[ PQ = 8 \times \frac{1}{2} \]
\[ = 4 \text{ cm} \]
\[ \tan \angle VQP = \frac{19.1833}{4} \]
\[ \angle VQP = 78.2^\circ \]

13. (a) \[ \angle BCD + 105^\circ = 180^\circ \text{ (opp. } \angle \text{s, cyclic quad.)} \]
\[ \angle BCD = 75^\circ \]
In \( \triangle ABCD \), by sine formula,
\[ \frac{16}{\sin \angle BDC} = \frac{18}{\sin 75^\circ} \]
\[ \sin \angle BDC = \frac{16 \sin 75^\circ}{18} = 0.8586 \]
\[ \angle BDC = 59.1598^\circ \]
\[ \angle DBC + 59.1598^\circ + 75^\circ = 180^\circ \text{ ( } \angle \text{ sum of } \triangle \text{)} \]
\[ \angle DBC = 45.8402^\circ \]
\[ = 45.8^\circ \text{ (correct to 3 significant figures)} \]
13. (b) Area of \( \triangle BCD \) \( \frac{1}{2} \times 18 \times 16 \sin 45.8402^\circ = 103.31 \) 
\( = 103 \text{ cm}^2 \) \( \text{(correct to 3 significant figures)} \)

14. (a) When \( x = 0^\circ, \ y = 2 \) 
\[ 2 = k \times \cos 0^\circ \]
\[ 2 = k \times 1 \]
\[ k = 2 \]

(b) (i) \( y = 2 \cos x \)
Equation of the new graph is \( y = 2 \cos x + 1 \)
Correct sketch of the new graph

(ii) Maximum value \( = 2 \times 1 + 1 \)
\( = 3 \)
Minimum value \( = 2 \times (-1) + 1 \)
\( = -1 \)

15. (a) \( \angle OPA = 90^\circ \) \( \text{(tangent perp. to radius)} \)
\[ \therefore \angle OPA = \angle SCA \]
\[ \angle PAO = \angle CAS \] \( \text{(tangent properties)} \)
\[ \angle AOP = \angle ASC \] \( \text{(\angle sum of \triangle)} \)
\[ \therefore \triangle APO \sim \triangle ACS \] \( \text{(equiangular)} \)
(b) \[ BA^2 = 18^2 + 24^2 \]
\[ = 900 \]

\[ BA = \sqrt{900} = 30 \text{ cm} \]

Let \( r \) be the radius of the circle,

\[ OQ = OR = r \]
\[ QC = OR = r \]
\[ CR = OQ = r \]

\[ \therefore \ AR = 24 - r \]
\[ AP = AR \quad \text{(tangent properties)} \]
\[ = 24 - r \quad \text{.................(1)} \]

\[ BQ = 18 - r \]
\[ BP = BQ \quad \text{(tangent properties)} \]
\[ = 18 - r \quad \text{.................(2)} \]

From (1) and (2),

\[ (18 - r) + (24 - r) = 30 \]
\[ 2r = 12 \]
\[ r = 6 \text{ cm} \]

\[ \therefore \text{ The radius of the circle is } 6 \text{ cm.} \]

(c) \[ \therefore \triangle APO \sim \triangle ACS \]

\[ \frac{AP}{PO} = \frac{AC}{CS} \]
\[ \frac{24 - 6}{6} = \frac{24}{CS} \]
\[ CS = 8 \text{ cm} \]
(a) Cost of producing a T-shirt = \( \frac{3000}{100} \) = $30

(b) Let \( P = c + \frac{k}{N} \) where \( c, k \) are constants.

When \( N = 150, P = \frac{3750}{150} = 25 \)

\[
\begin{align*}
30 &= c + \frac{k}{100} \\
25 &= c + \frac{k}{150}
\end{align*}
\]

(1) – (2):

\[
30 - 25 = \frac{k}{100} - \frac{k}{150}
\]

\[
5 = \frac{3k}{300} - \frac{2k}{300}
\]

\[
5 = \frac{k}{300}
\]

\[ k = 1500 \]

Substituting \( k = 1500 \) into (1),

\[
30 = c + \frac{1500}{100}
\]

\[ c = 15 \]

\[ \therefore P = 15 + \frac{1500}{N} \]

(c) (i) Profit per T-shirt = $2500 \div 200 = $12.5

\[ P = 15 + \frac{1500}{200} = 22.5 \]

\[ \therefore \text{Selling price of a T-shirt} = 22.5 + 12.5 = 35 \]

(ii) Percentage profit \[ \frac{12.5}{22.5} \times 100\% = 55.6\% \]

(d) To avoid making any loss, \( P \leq 18 \).

\[ 15 + \frac{1500}{N} \leq 18 \]

\[ \frac{1500}{N} \leq 3 \]

\[ N \geq \frac{1500}{3} = 500 \]

\[ \therefore \text{At least 500 T-shirts should be produced and sold.} \]
17.

(a) Consider $\triangle ADC$, by the sine formula,

$$\frac{AD}{\sin(180^\circ - 95^\circ - 50^\circ)} = \frac{24}{\sin 50^\circ}$$

$$AD \approx 17.9700 \text{ cm}$$

$$= 18.0 \text{ cm}$$

(b)(i) Consider $\triangle BAD$, by the cosine formula,

$$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos 55^\circ$$

$$BD \approx 16.5952 \text{ cm}$$

$$= 16.6 \text{ cm (corr. to 3 sig.fig.)}$$

(ii)

Let $F$ be the foot of perpendicular from $B$ to $AC$ as well as the foot of perpendicular from $D$ to $AC$.

The required angle is $\angle BFD$.

In $\triangle DAF$,

$$\sin \angle DAF = \frac{DF}{AD}$$

$$\sin 50^\circ = \frac{DF}{17.9700}$$

$$\therefore DF \approx 13.7658 \text{ cm}$$

Since $ABCD$ is a kite,

$$BF = DF \approx 13.7658 \text{ cm}$$

Consider $\triangle BFD$, by the cosine formula,

$$\cos \angle BFD = \frac{BF^2 + DF^2 - BD^2}{2(BF)(DF)}$$

$$\therefore \angle BFD = 74.1^\circ \text{ (corr. to 3 sig.fig.)}$$
Let $S$ be the point on $PQ$ such that $OS \perp PQ$ and $RS \perp PQ$.

Area of $\triangle OPQ = \frac{1}{2}(PQ)(OS)$

Area of $\triangle RPQ = \frac{1}{2}(PQ)(RS)$

\[
\frac{\text{Area of } \triangle OPQ}{\text{Area of } \triangle RPQ} = \frac{\frac{1}{2}(PQ)(OS)}{\frac{1}{2}(PQ)(RS)} = \frac{OS}{RS} = \cos \theta
\]

(ii)

Let $M$ be the foot of perpendicular from $C$ to $AB$ and $\angle CME = \phi$

\[
\frac{1}{2}(AB)(CM) = 12
\]

\[
\frac{1}{2}(6)(CM) = 12
\]

$CM = 4$

\[
\sin \phi = \frac{CE}{CM} = \frac{2}{4} = \frac{1}{2}
\]

\[
\therefore \phi = 30^\circ
\]

From (c)(i), area of $\triangle EAB = (\text{area of } \triangle CAB) \cos \phi$

\[
= 12 \cos 30^\circ
\]

\[
= 6\sqrt{3} \text{ m}^2
\]

\[
\therefore \text{the area of the shadow is } 6\sqrt{3} \text{ m}^2
\]