Answers of Maths Mock Exam 2008 Paper 1

1. \( a^n b^2 \)
2. \( a = \frac{5x^2 - u}{4 + x^2} \)
3. (a) \( 2(2x + y)(2x - y) \) (b) \( 6k - 1)(k^2 - 5) \)
4. (a) \( a : b : c = 4 : 2 : 3 \) (b) 36
5. $3795
6. \( -5\sqrt{2} \) or \(-12\sqrt{2}/7 \)
7. \( \frac{1}{3} \leq x \leq 2 \)
8. \( 550cm^2 \)
9. \( x = 30^\circ \), \( y = 120^\circ \)
10. (a)(i) 11 (ii) 6 – 10 (b)(i) 14.5 (ii) 48%

Marking Scheme for Maths Mock Exam 2008 Paper 1

1. \( \frac{a^3 b^2}{\sqrt{a^2 b^2}} \times \frac{b^n a^4}{\sqrt{a^n}} \)
   \( = \frac{a^{3/2} b^{5/2} a^4}{a^2 b^2} \)
   \( = \frac{a^{1/2} b^{11/2}}{a^2 b^2} \)
   \( = \frac{a^{1/2}}{a^{2}} b^{5/2} b^{-2} \)

3. (a) \( 8x^2 - 2y^2 = 2(4x^2 - y^2) \)
   \( = 2((2x + y)(2x - y)) \)
   \( = 2(2x + y)(2x - y) \)
   (b) by making use the result in (a), we put \( x = k - 2 \) and \( y = k + 1 \), we have 
   \( 8(k - 2)^2 - 2(k + 1)^2 = 2(2(k - 2) + (k + 1))(2(k - 2) - (k + 1) \)
   \( = 2(3k - 3)(k - 5) \)
   \( = 6(k - 1)(k - 5) \)

2. \( (5 - a)x^2 = 4a + u \)
   \( 5x^2 - ax^2 = 4a + u \)
   \( a(4 + x^2) = 5x^2 - u \)
   \( a = \frac{5x^2 - u}{4 + x^2} \)

4. (a) \( 3a = 6b = 4c \)
   \( \frac{3a}{12} = \frac{6b}{12} = \frac{4c}{12} \)

5. The amount after 4 years 
   \( = $30000(1 + 1.5\%)^4 \)
   \( = $33794.76 \)
   \( \therefore \) the interest = $33794.76 – $30000
   \( = $3795 correct to nearest dollar. \)

11. (a) \( a = 4, b = -1 \)
    (b) (i) proof (ii) \( x = 0 \) or \( x = \frac{-5 + \sqrt{7}}{2} \)

12. (a)(i) \( \frac{39}{248} \) (ii) \( \frac{153}{496} \)
    (b)(i) \( \frac{26}{57} \) (ii) \( \frac{6}{247} \)

13. (a) \( a = 1, b = 3 \)
    (b) \( y = 2x - 1 \)
    (c) \( y = -2x + 3 \)
    (d) \( y = -2x + 3 \)

14. (a) \( A = x(8 - x) \)
    (b) max \( \Delta = 16 \) when \( x = 4 \)
    (c) \( P = 3.6\Delta - 0.2\Delta^2 \)
    (d) max profit is $16.2 when \( \Delta = 9 \)

15. (a) \( VC = 30.5m \) \( AC = 75.5m \)
    (b)(i) \( 36.2^\circ \)
    (ii) \( 47.7 m \)

16. (a) \( 2x + 5y - 22 = 0 \)
    (b)(i) \( A'(2, 6), B'(-4, 1) \)
    (ii) \( (3, \frac{1}{2}) \)
    (c)(i) \( 4x + 10y - 23 = 0 \)
    (ii) \( x^2 + y^2 + \frac{31}{11}x - \frac{63}{11} = 0 \)
    \( = 0 / 11x^2 + 11y^2 + 31x - 63y = 0 \)

17. (a)(i) proof (ii) proof
    (b)(i) 6.94cm (ii) \( (20 + 10\sqrt{3})cm \)

(e) \( 1.4 \times 6.6cm^2 \)
6. Given \( \cos \theta = \frac{3}{4} \),
then \( \sin \theta = \frac{\sqrt{7}}{4} \),
and \( \tan \theta = \frac{\sqrt{7}}{3} \)
\( \therefore \theta \) in Quadrant IV.
\[ \therefore \cos^2 \theta + \sin^2 \theta = 1 \]
\[ \tan \theta - \sin \theta = \frac{\sqrt{7}}{12} \]
\[ \therefore \text{The result is valid.} \]

9. \( \angle BCA = 90^\circ \) (\( \angle \) in semi-circle)
\( x + \angle BCA = 120^\circ \) (ext. \( \angle \) of \( \Delta \))
\[ \therefore x = 120^\circ - 90^\circ = 30^\circ \]
y = 120° (ext. \( \angle \), cyclic quad.)

10. (a) (i) \( x = 50 - (6 + 12 + 8 + 10 + 3) \)
\[ x = 11 \]

(ii) The modal class is 6 – 10

(b) (i) The mean = \[ \frac{\sum f_i x_i}{\sum f_i} \]
\[ = \frac{2.5\times6+3\times8+13\times10+23\times11+28\times11}{50} \]
\[ = 14.54 \]
\[ = 14.5 \text{ (3 sig. fig.)} \]

(ii) passing percentage = \[ \frac{11+10+3}{50} \times 100\% \]
\[ = 48\% \]

7. \((x-2)(x+4) \leq 0 \) and \( x > \frac{1}{3} \)
\[ -4 \leq x \leq 2 \text{ and } x > \frac{1}{3} \]

Combine the result we get
\[ \frac{1}{3} \leq x \leq 2 \]

8. Total surface area of the solid
\[ = \pi(5)^2 + 2\pi(5)(10) + 2\pi(5)^2 \]
\[ = (25 + 100 + 50)\pi \]
\[ = 175\pi \text{ cm}^2 \]
\[ = 550\text{ cm}^2 \text{ (to nearest cm}^2) \]

11. (a) \( \because x - 1 \text{ is factor of } f(x) \)
\[ f(1) = 0 \]
\[ \therefore 2(1)^3 + a(1)^2 - 5(1) + b = 0 \]
\[ a + b = 3 \ldots \ldots (1) \]

Also, when \( f(x) \) is divided by \( x + 2 \), the remainder is 9
\[ \therefore f(-2) = 9 \]
\[ 2(-2)^3 + a(-2)^2 - 5(-2) + b = 9 \]
\[ 4a + b = 15 \ldots \ldots (2) \]
\[ (2) - (1), 3a = 12 \]
\[ a = 4 \]
Sub. \( a = 4 \) into (1)
\[ b = -1 \]

12. (a) (i) \( P(\text{Both are female}) \)
\[ = \frac{13}{32} \times \frac{12}{31} \]
\[ = \frac{153}{992} \]
\[ = 0.154 \]

(ii) \( P(\text{Both are classified as driving under the influence of alcohol}) \)
\[ = \frac{18}{32} \times \frac{17}{31} \]
\[ = \frac{312}{992} \]
\[ = 0.314 \]

(b) (i) \( P(\text{Only one pass the test}) \)
\[ = \frac{13}{32} \times \frac{17}{31} \]
\[ = \frac{221}{992} \]
\[ = 0.222 \]
13. (a) When \( x \) tends to infinity, the value of function tends to 3, while \( 2^{-x} \) tends to 0 as \( n \) tends to infinity, so \( b = 3 \). The \( y \)-intercept of the function is 5, i.e. \( 2^{3} - 2x + b + 1 = 0 \). From the result of (a), the equation can be written as \( 2^{x+1} - 2x + 3 + 1 = 0 \), \( 2^{x+1} + 3 = 2x - 1 \). The required equation is \( y = 2x - 1 \) 1A

(b) The transformation is equivalent to \( y = -f(x) \) for given \( y = f(x) \), i.e. \( y = -2^{x+1} - 3 \). The required equation is \( y = 2^{x+1} + 3 + 6 \). The new equation is formed by move the graph 6 units upward, \( y = 2^{x+1} + 3 \). 1A

(c) Let \( P = k_{1}A^2 + k_{2}A^3 \), \( k_1, k_2 \neq 0 \). \[ 6.4 = 2^2k_1 + 2k_2 \]
\[ 16 = 10^2k_1 + 10k_2 \]
i.e. \[ 4k_1 + 2k_2 = 6.4 \]
\[ 100k_1 + 10k_2 = 16 \]
On solving, we have \( k_1 = -0.2, k_2 = 3.6 \). \( \therefore P = 3.64 - 0.2A^2 \). 1A

(d) \( P = 3.64 - 0.2A^2 \)
\[ P = -0.2(A^2 - 18.4) \]
\[ = -0.2(A^2 - 18.4 + 81) + 16.2 \]
\[ = -0.2(A - 9)^2 + 16.2 \]
\( \therefore \) the maximum profit is $16.2 when \( A = 9 \) 2A

(e) By adding the line \( A = 9 \), we have \( x = 1.4 \) or \( x = 6.6 \), both yields the dimension as 1.4x6.6cm² such that \( P \) attains the maximum. 1A

15 (a) \( \angle VBC = 28^\circ \) (alt \( \angle s // line \))
\( VC = VB \sin 28^\circ \)
\[ = 65 \sin 28^\circ \]
\[ = 30.5m \text{ (3 sig. fig.)} \] 1A

(b) \( \angle DAC = \angle DCA = \frac{180^\circ - 130^\circ}{2} = 25^\circ \)
By the Sine Formula, 1A

16 (a) The equation of \( AB \) is \( \frac{y - 4}{x - 1} = \frac{2 - 4}{6 - 1} \) 1M
i.e. \( 2x + 5y - 22 = 0 \) 1A

(b) (i) \( A'(-2, 6), B'(-4, 1) \) 1A + 1A
(ii) \( \because \text{mid-point of } A'B' \text{ is } \left( \frac{2 - 4}{2}, \frac{6 + 1}{2} \right) = (-3, \frac{7}{2}) \) 1A

(c) (i) By (a), the slope of \( AB \) is \( \frac{2}{5} \)
\( \therefore \) \( AB \) is perpendicular bisector of \( A'B' \) 1M
(ii) The slope of the perpendicular bisector of \( A'B' \) is \( -\frac{2}{5} \) 1M
By (b)(ii), the ⊥ bisector passes through
\((-3, \frac{7}{2})\),
\[
\begin{align*}
\frac{y - \frac{7}{2}}{x + 3} &= \frac{2}{5} \\
\Rightarrow 4x + 10y - 23 &= 0
\end{align*}
\]
∴ The equation of the perpendicular bisector of A'B' is
\[4x + 10y - 23 = 0\]
(ii) Let the equation of the required circumcircle be
\[x^2 + y^2 + Dx + Ey = 0.\]
Substitute A'(-2, 6) and B'(-4, 1) into this equation respectively,
\[
\begin{align*}
(-2)^2 + 6^2 - 2D + 6E &= 0 \\
(-4)^2 + 1^2 - 4D + E &= 0
\end{align*}
\]
\[
\begin{align*}
2D - 6E &= 40 \\
4D - E &= 17
\end{align*}
\]
on solving, we have D = \(\frac{31}{11}\), E = \(\frac{63}{11}\).
Thus, the equation of the circumcircle of \(\Delta OA'B'\) is
\[x^2 + y^2 + \frac{31}{11}x - \frac{63}{11}y = 0.\]
OR
\[11x^2 + 11y^2 + 31x - 63y = 0\]
17 (a) \(\angle P_3P_4 = d \sin \theta \text{ cm}\)
\[
\angle P_3P_4P_5 = 180^\circ - 90^\circ - \theta \text{ ( \sum of } 1M \text{ angles)}
\]
\[
\begin{align*}
\angle P_3P_4P_5 &= 90^\circ - \theta \\
P_3P_5 &= P_3P_4 \sin \angle P_3P_4P_5 \\
&= (d \sin \theta)(\sin \theta) \\
&= d \sin^2 \theta \\
\angle P_5P_3P_4 &= 0 \text{ (corr. } \angle s, P_2P_4 \parallel P_4P_3) \\
P_2P_6 &= P_5P_3 \sin \angle P_5P_3P_6
\end{align*}
\]
\[
\sum P_3P_6 = P_3P_5 \sin \angle P_3P_5P_6
\]
\[= d \sin^3 \theta \text{ cm} \quad 1A
\]
\[
P_6P_4 = P_4P_5 \cos \theta = dsin^2 \theta \cos \theta \text{ cm} \\
P_8P_6 = P_6P_7 \cos \theta = dsin^4 \theta \cos \theta \text{ cm} \\
P_{10}P_8 = P_8P_9 \cos \theta = dsin^6 \theta \cos \theta \text{ cm}
\]
\[
\begin{align*}
P_6P_4 &= P_8P_6 = \ldots = \sin^2 \theta \\
P_4P_6 &= P_{10}P_8 = \ldots = \sin^4 \theta \\
P_6P_8 &= P_{10}P_6 = \ldots = \sin^6 \theta
\end{align*}
\]
\[
\begin{align*}
\therefore P_4P_6, P_6P_8, P_8P_{10}, \ldots \text{ form a geometric sequence.}
\end{align*}
\]
(b) (i) \[P_6P_8 + P_8P_{10} + \ldots + P_{20}P_{22} = \frac{d(\sin^2 \theta \cos \theta)(1 - \sin 18 \theta)}{1 - \sin^2 \theta} \text{ cm} \quad 1M\]
\[
\begin{align*}
&= \frac{5(\frac{1}{1 - \sin 60^\circ})}{1 - \sin^2 \theta} \\
&= \frac{5(\frac{1}{1 - \frac{\sqrt{3}}{2}})}{1 - \sin 60^\circ} \text{ cm} \\
&= \frac{10}{2 - \sqrt{3}} \\
&= \frac{10 \times 2 + \sqrt{3}}{2 - \sqrt{3} \times 2 + \sqrt{3}} \\
&= (20 + 10\sqrt{3}) \text{ cm}
\end{align*}
\]
\[
\sum P_4P_6 = P_6P_8 = \ldots = \sin 2 \theta \text{ cm} \quad 1A
\]
\[
\begin{align*}
P_4P_6 &= P_8P_10 = \ldots = \sin 2 \theta \\
P_6P_8 &= P_{10}P_6 = \ldots = \sin 4 \theta \\
P_8P_{10} &= P_6P_8 = \ldots = \sin 6 \theta
\end{align*}
\]
\[
\begin{align*}
\therefore P_4P_6, P_6P_8, P_8P_{10}, \ldots \text{ form a geometric sequence.}
\end{align*}
\]